Brush-Up Maths for Data Science (2025)

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Sequences, Sums & Products

Mathematical expressions that often appear:

- Sequences: $1, 2, 3, 4, 5, \ldots$
- Sums: $1 + 2 + 3 + 4 + \ldots$
- Products: $1 \cdot 2 \cdot 3 \cdot 4 \cdot \ldots$

Specialized notation has been developed to describe these concisely.

In the following, we will look at:

- How sequences are written
- How sums are expressed using sigma (Σ) notation
- How products are expressed using pi (Π) notation

Sequences

A sequence is an ordered list of numbers, where terms are written with subscripts:

$$a_0, a_1, a_2, a_3, a_4, a_5, \ldots$$

In this context, note:

- For sequences order matters (unlike in sets)
- The entire sequence is often written as:

$$\{a_n\}_{n=0}^{\infty} \quad \text{or} \quad \{a_n\}$$

The number in the subscript is called the index (plural: indices).

Sums

Given a sequence $\{a_n\}$ and integers m, p with $m \leq p$, the sum is written:

$$\sum_{n=m}^{p} a_n = a_m + a_{m+1} + \cdots + a_p$$

In this context, note:

- The variable n is called the index of summation
- The number m is called the lower limit of summation
- The number p is called the upper limit of summation

Sums

- Examples

Expand the following, i.e., write out (some) terms of:

$$\sum_{n=1}^{100}(3+4n) =$$

$$\sum_{n=1}^{p}\left(\frac{1}{2}\right)^{n} =$$

Evaluate the following sums:

$$\sum_{g=4}^{9}(5g+3) =$$

$$\sum_{k=1}^{3}\frac{7}{10^k} =$$

Use sum notation to rewrite the following:

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{50} =$$

$$1-2+3-4+5-6+7-8 =$$

Sums

- Another Example

Summation notation allows us to define polynomials as functions of the form:

$$\begin{aligned} f(x) &= a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \\ &= \sum_{k=0}^{n} a_k x^k \end{aligned}$$

Here:

- n is a non-negative integer (the degree of the polynomial)
- $a_n, a_{n-1}, \ldots, a_0$ are real constants
- $a_n \neq 0$ if $n > 0$

Products

Given a sequence $\{a_n\}$ and integers m, p with $m \leq p$, the product is written:

$$\prod_{n=m}^{p} a_n = a_m \cdot a_{m+1} \cdot \cdots \cdot a_p$$

In this context, note:

- The variable n is called the index of multiplication
- The number m is called the lower limit of the product
- The number p is called the upper limit of the product

Products

- Examples

Expand the following, i.e., write out (some) terms of:

$$\prod_{n=1}^{4} n =$$

$$\prod_{n=0}^{p} (2+3n) =$$

Use product notation to rewrite the following:

$$2 \cdot 4 \cdot 6 \cdots 2p =$$

$$1 \cdot 3 \cdot 5 \cdots (2p-1) =$$

Evaluate the following products:

$$\prod_{n=1}^{p} 5 =$$

$$\prod_{n=1}^{3} (n+1) =$$

Exercise Set

- Part 1

Expand the following sums/products. That is, write out (some of) the terms:

1. $\sum_{k=0}^{n} 2^k$
2. $\prod_{k=2}^{100} \frac{k^2}{(k^2-1)}$

Use summation/product notation to rewrite the following:

3. $2+4+6+8+\cdots+2n$
4. $(\frac{1}{2})(\frac{2}{3})(\frac{3}{4})\cdots(\frac{100}{101})$

Evaluate the following sums:

5. $\sum_{k=1}^{3} \frac{7}{10^k}$
6. $\sum_{k=0}^{2}(3k-5)x^k$
7. $\sum_{n=1}^{100}(-1)^n$