Mathematics Brush-Up for Data Science

Lecture Slides 2: Set Operations

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Set Operations

Understanding how sets relate and combine is fundamental when comparing collections and forming new sets.

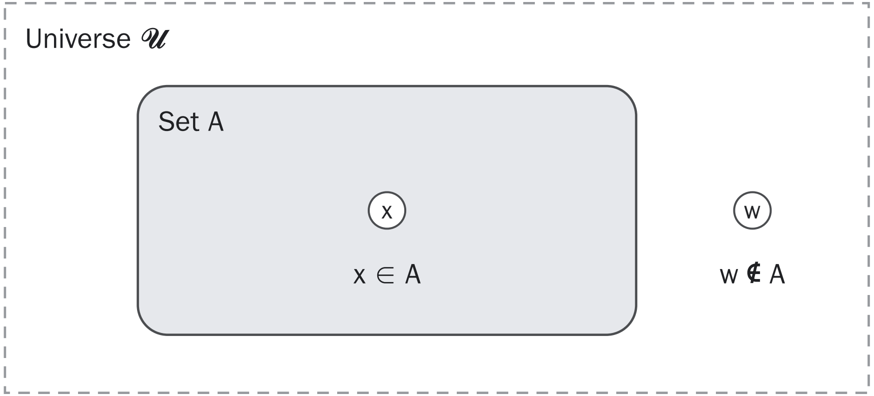
In this section, we introduce the following key concepts and their common symbols:

- **Universe** ($\mathcal{U}$)
- **Set difference** ($\setminus$)
- **Union** ($\cup$)
- **Intersection** ($\cap$)

These concepts establish the context and describe what sets exclude, combine, and share.

The Universe

- Definition



The universe ($\mathcal{U}$) is the set containing all elements under consideration in a given context.

- All sets under discussion are considered subsets of $\mathcal{U}$

The Universe

- Examples

If we let the universe be the set of all lowercase English letters:

$$\mathcal{U} = \{\mathrm{a}, \mathrm{b}, \mathrm{c}, \ldots, \mathrm{z}\}$$

Then it gives context to the following sets:

- $A = \{x \mid x \text{ is a vowel}\}$
- $B = \{x \mid x \text{ is a consonant}\}$

If we let the universe be the set of real numbers:

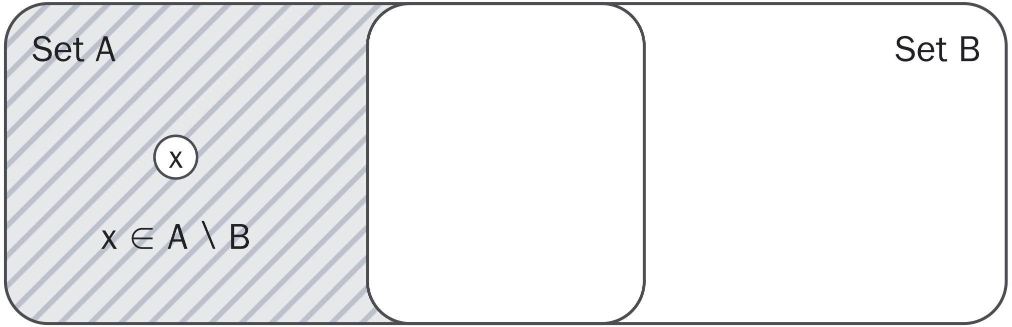
$$\mathcal{U} = \mathbb{R}$$

then it gives context to:

- The numbers strictly between 0 and 1: $(0, 1)$
- The numbers between 2 and 5, including both: $[2, 5]$
- The real numbers greater than 3: $(3, \infty)$

Set Difference

- Definition



The difference of A and B, written $A \setminus B$, is:

$$A \setminus B = \{x \in A \mid x \notin B\}$$

That is, we obtain the elements in A but not in B.

The notation $A - B$ is also used; here, $\setminus$ distinguishes set difference from numerical subtraction.

Set Difference

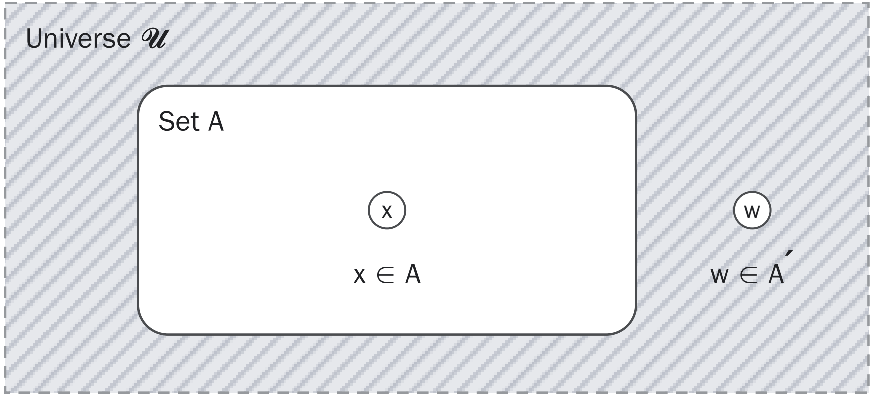
- Examples

In each case, compute $A \setminus B$:

1. Let $A = \{1, 2, 3\}$ and $B = \{2, 4\}$
2. Let $A = \{\text{apple}, \text{banana}, \text{pear}\}$ and $B = \{\text{pear}, \text{peach}\}$
3. Let $A = \mathbb{R}$ and $B = \mathbb{Q}$
4. Let $A = \{1, 2\}$ and $B = \{1, 2\}$
5. Let $A = [0, 1]$ and $B = (0, 1)$

The Complement

- Definition



The complement of A, denoted A' or A^c, is:

$$A' = A^c = \mathcal{U} \setminus A$$

All elements in the universe $\mathcal{U}$ that are not in A

The Complement

- Examples

In each case, given a universe $\mathcal{U}$, determine A':

1. Let $\mathcal{U} = \{1, 2, 3, 4, 5\}$ and $A = \{1, 2, 3\}$
2. Let $\mathcal{U}$ be a deck of cards and A the set of spades
3. If $\mathcal{U} = \mathbb{R}$ and $A = \{x \in \mathbb{R} \mid x > 10\}$.

Exercise Round 1

- The Universe, Set Difference & Complement

Let

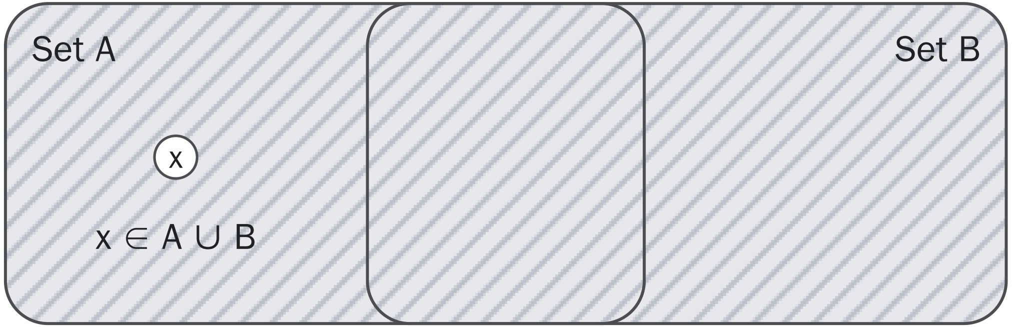
$$\begin{aligned} \mathcal{U} &= \{1, 2, 3, 4, 5, 6, 7, 8\}, \\ A &= \{2, 4, 6\}, \\ B &= \{1, 2, 3, 4\}. \end{aligned}$$

Find each of the following sets.

1. $A \setminus B$
2. $B \setminus A$
3. A'
4. B'
5. $(A \setminus B) \setminus A'$

The Union

- Definition



The union of A and B, written $A \cup B$, is:

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

That is, we obtain all elements in A, in B, or in both.

Here, “or” is inclusive: an element may belong to either set or to both sets.

The Union

- Examples

In each case, compute $A \cup B$:

1. Let $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$
2. Let $A = \{\text{red}, \text{blue}\}$ and $B = \{\text{green}, \text{blue}\}$
3. Let $A = \{1, 2\}$ and $B = \{3, 4\}$

The Intersection

- Definition

The intersection of A and B, written $A \cap B$, is:

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

That is, we obtain the elements that A and B have in common.

The Intersection

- Examples

In each case, compute $A \cap B$:

1. Let $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$
2. Let $A = \{\text{red}, \text{blue}\}$ and $B = \{\text{green}, \text{blue}\}$
3. Let $A = \{1, 2\}$ and $B = \{3, 4\}$

Exercise Round 2

- Union & Intersection

For each pair of sets, write the result in roster form. Let

$$
\begin{aligned}
A &= \{0, 2, 4, 6\}, & B &= \{2, 3, 4, 5\}, \\
C &= \{\text{cat}, \text{dog}, \text{bird}\}, & D &= \{\text{dog}, \text{fish}\}, \\
E &= \{1, 3, 5\}, & F &= \{2, 4, 6\}.
\end{aligned}
$$

Unions

1. $A \cup B$

2. $C \cup D$

3. $E \cup F$

Intersections

4. $A \cap B$

5. $C \cap D$

6. $E \cap F$

Union & Intersection

- Basic Laws

Let A, B, and C be subsets of the universe $\mathcal{U}$.

Law	Union	Intersection
Commutative	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Associative	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
Distributive	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity	$A \cup \emptyset = A$	$A \cap \mathcal{U} = A$

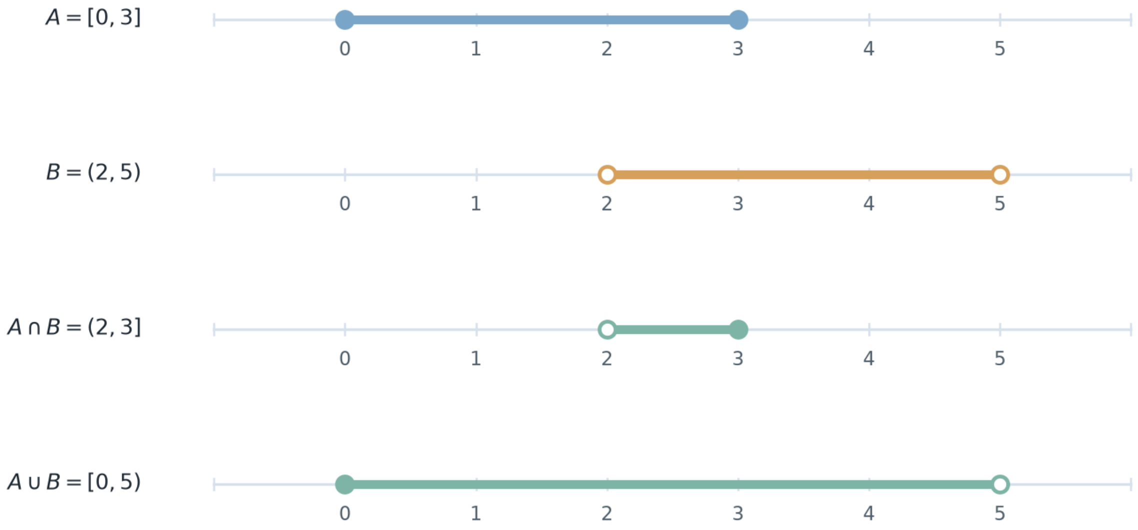
These laws let us reorder, regroup, and simplify expressions without changing the set they describe.

Union & Intersection

- Operations on Intervals

Let $A = [0, 3]$ and $B = (2, 5)$.

Compute $A \cap B$ and $A \cup B$.

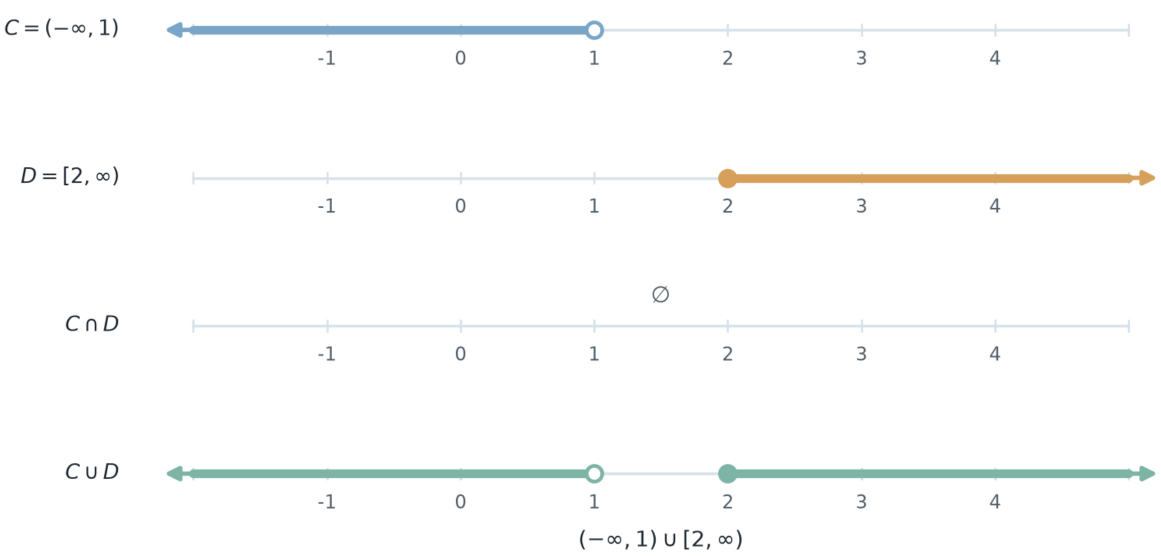


Union & Intersection

- Separated Intervals

The intervals $C = (-\infty, 1)$ and $D = [2, \infty)$ are separated: they do not overlap.

Compute $C \cap D$ and $C \cup D$.



Exercise Round 3

- Interval Operations

Find the union and intersection of each pair of sets.

1. $E = (-2, 4] \quad \text{and} \quad F = [1, 6)$
2. $G = (-\infty, 0) \quad \text{and} \quad H = [0, \infty)$
3. $I = [1, 3] \quad \text{and} \quad J = [4, 6]$

Additional Set Operations

A few other foundational set operations are commonly used in mathematics.

The table below provides a brief overview.

Symbol	Operation	Description
$A \times B$	Cartesian product of A and B	The set of all ordered pairs (a, b) where $a \in A$ and $b \in B$
$\mathcal{P}(A)$	The power set	The set of all subsets of A, including the empty set ($\emptyset$) and A itself